MATHEMATICAL ANALYSIS APPLIED TO SCHOOL ATTENDANCE AREAS

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Abstract—The objective of this paper is to develop a generalized mathematical model of pupil assignment within school districts. This model can then be used to examine various policies of student integration. Proposed bussing schemes, school location policies, educational parks, attendance boundaries, etc., can be tested for cost, travel time or other measures of efficiency. Extension to other areas of educational planning is feasible.

Mathematical programming techniques are used to assign resources (say school children) to facilities (say schools) subject to restrictions on facilities (say capacity limits) and resources (say a maximum travel time or a desirable range of school "mixtures") so that a measure of performance, the "objective function" (say total cost or total time of travel) is optimized. The model is intended to allow examination of a wide range of objective functions and system constraints.

It is customary, in any mention of school segregation today, to refer to the 1954 Supreme Court decision as though this were the only starting point, one might almost say justification, for the various efforts now being made to achieve "racial balance in the schools." Sober consideration of the domestic and foreign events and currents of thought in the intervening 13 yrs, however, leads us to understand that a multitude of other considerations would in all likelihood have led us to the same efforts and the same triumphs and frustrations in these endeavors as we are undergoing now.

Foremost among these currents of thought are purely economic considerations. The availability and development of human resources are often reflected in our calculations concerning the economy of the country as a whole. When a new program like Medicare is implemented, there is a sudden flurry of concern about whether there will be enough nurses. When a particular state is running low on welfare funds, there is a public debate on whether it is cheaper to pay welfare assistance or to train the recipients so that they may earn their own livings. Such piecemeal expressions of concern miss the main point: more than 15 per cent of our population are not vouchsafed the opportunity to contribute their fair share to their own or the general prosperity. Putting it another way: We are losing 15 per cent of what should be our best brains in every graduating class. We have, in fact, a "brain drain" of our own. It does not take the form of training graduates to a high pitch of expertise and then losing them by emigration. Instead, we are losing them through neglect and lack of development to their fullest potential.

In this, we resemble (oddly enough) foreign countries whose class or caste systems we tend to disparage. We ourselves may well be face-to-face with a system in which opportunity for advancement and earning ability is vastly superior for a segment of our
population. On purely economic grounds, such a situation leads to an instability that we are not prepared to tolerate, let alone pass on to future generations.

What then of modern sociological concerns, where indeed they can be distinguished from the purely economic? The Supreme Court decision of 1954 has itself been accused of acting on sociological rather than legal grounds, but this is open to grave doubt. In the North and West, school segregation evolved logically from segregation in housing, given the so-called tradition of the neighborhood school. The court decision did not define these basic causes of segregation and act to remedy them, and none of the efforts made as a result of the decision do in fact remedy them.

The social environment for school segregation was created by residence patterns, themselves a reflection of racial prejudice on the one hand and a benevolent centralized attempt at home financing on the other. We have no clear idea of what urban development might have been like if the generous financial programs to foster home ownership in the thirties and for returning veterans in the late forties had not been instituted (just as we cannot now imagine the whole picture of higher education in this country without the G.I. Bill and its successors). The fact remains that the loan programs, administered and applied by bankers on sound principles of creditworthiness, created model suburbs and districts which were far more highly segregated than the parent cities had been. The void left in the urban centers by this accelerated and encouraged flight to suburbia has been filled by “immigrant” minority groups, leading to the creation of larger and less influential ghetto areas than this country has ever known. Preferring to make loans for new construction rather than for the remodeling of older homes may, for example, have been a good principle of sound management of taxpayers’ money. Nevertheless, the social costs of such a preference have been far-reaching and may indeed prove to be exorbitant. The housing patterns created by the FHA and the public housing agencies are nearly all segregated patterns, leading in turn to segregated schools. An attempt to stem this trend was the issuance in 1962 of a specific directive to FHA officials on improving the flow of loans to minority groups. The unimpressive results in the last 5 yrs still showed that fewer than 3 per cent of the total number of loans were made to Negroes. Again it did not need the legal spur of 1954 to make it clear that the polarization of rich and poor, white and non-white, into suburbia and city center will lead to even greater instability if continued unchecked.

It has become common to talk of people or children as “deprived” or the opposite. What we must realize, and make sufficiently clear to the world at large, is that this situation DEPRIVES us all. It is we who are being deprived of more than 15 per cent of the best brains in our land and it is we who are deprived of a stable, properly developed, economy and society. Since the Supreme Court decision forced the issue of equality of opportunity into the public eye, the U.S. public educational system has been the main battleground for the achievement of this goal. It is not too far-reaching to say that the public conscience and the long-term public weal are being determined here and now by the nation’s school system.

While the economic and sociological considerations just outlined may be cogent, they are often regarded as too qualitative a basis for action. The legal basis, at least, is felt to be a “fact”. It therefore comes as a rude shock to realize that the Supreme Court did not provide us with many facts to go on. The terms “segregation” or “desegregation” are not defined, there is no approved measure of success or failure in efforts to achieve a balanced pupil community. Common sense indicates that the ethnic composition of a school population must somehow reflect the ethnic composition of the population as a
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whole. Such a criterion then immediately raises more questions, such as: "How wide a difference between the two compositions is tolerable?", "What if the school districts were deliberately engineered along ethnic lines?" etc.

The search for means of integrating racially and culturally divergent groups in individual school systems has become a vexing question and a painful process in almost all parts of the country. (One is still amazed at the legal and political shifts to which some areas have resorted in order to avoid any changes whatsoever. Even areas with "the best will in the world," however, have frequently undergone severe upheavals and logical gymnastics without necessarily showing much in the way of successful results). Additional legislation and interpretation of existing legislation, both state and federal, have been necessary to assist in the implementation of school integration goals. And so, in spite of the millions of words and man-hours expended on the subject within the last two decades, and principles of law and administration notwithstanding, the bulk of our schools today remain segregated.

It is therefore surely a relief to turn to any quantitative measures that can help us solve the problems we have been set. Before describing in detail the analytic methods of mathematical programming which can provide theoretically optimal solutions to problems of allocations and assignment, let us examine some other quantitative aids. Since segregated schools are caused by segregated housing, it is helpful to consider the Taeuber "segregation index"[1] which is a simple measure of the latter. The rationale of the index is that, if there were no forces working toward residential segregation in a city, any given neighborhood would have the same ethnic composition as the city as a whole, and a reading of zero would be obtained. At the other end of the scale, a reading of 100 is obtained when there is complete racial segregation. If one were concerned solely with integration in housing, the index would essentially give the percentage of minority group populations that need to be moved, and to where, to attain full integration. To obtain a similar aid to school integration, some derivative index is required, so as to take into account the age distribution of the populations under consideration. A city composed entirely of Negroes with young children and elderly white couples without children can well achieve zero on the index scale but never achieve school integration with existing boundaries. Clearly, this measure also depends to a great extent on the boundaries chosen to define "neighborhoods" and on several other judgemental factors, but it has a certain usefulness, especially when time-series are generated. The Tauebers[1] applied the measure to 207 American cities, using block-by-block data from the 1960 census. The index ranged from a low 60.4 in San Jose, California to a high of 98.1 in Fort Lauderdale, Florida. In general, it tends to range from 80-85 in the North and from 90-98 in the South.

The Taeuber index for Negro versus all other ethnic groups for San Francisco (1960 census) was 69.1 (reduced from 79.8 in 1950). The comparable index for San Francisco elementary schools in 1966 (based on the school attendance data issued by the San Francisco School District in 1967) was 67.4. Here, we are faced with the need for further quantitative investigation. Considering that school attendance is drawn from larger geographical units than a single city block, the figures suggest that school attendance areas had been drawn in a way that reflected racial residence patterns. In fact, the School District has recently sponsored a study to determine new pupil-to-school assignments that would reduce segregation of the several minority groups present in San Francisco.

This study[2] divided the attendance in elementary schools into four major categories, present in the following proportion:
The study worked with the figures in the right-hand column, making group (2) include Japanese (1.9 per cent) and Filipino (2.4 per cent) students. Since both the Japanese and Filipino populations are well-integrated by residence areas, our own analysis would have included them under "all others", as in the left-hand column. This would also have obviated the need for a grouping designated "Oriental" with a comparison group "White" as it is used in the study's report.

The study took about 9 months to complete and cost over $200,000. The research task force examined a large number of alternative pupil assignment systems, ranging from simple pair redistricting and school-pairing to school feeder systems (from elementary through high school) and changes in school patterns (4-4-4; 3-3-3-3; etc.). The quantitative measures used were a quadratic-quadriracial index (directly related to Taeuber's index) and costs (both capital and operating) for busses and schoolrooms. No optimizing procedure was considered for the allocations.

The resulting nine to twelve plans were presented at open School Board meetings. While many of the plans improved the integration index for the Negro and Oriental minorities, no plan made any significant improvement in the integration index of the Spanish-speaking minority. The report[2] stated that such improvements would be too costly, although no actual cost figures were given. Further, the integration measure used in evaluating alternative assignment patterns was "partially colorblind." A school is considered integrated if it has a Negro-Oriental mix or a White-Negro mix, or any such combination of pairs. One is led to suspect than an abhorrence of bussing, particularly of bussing "White" students, served as a greater constraint than either integration or cost. To quote from the report issued (Research Memorandum No. 7, p. 48):

"While cross-bussing under the D-2 pattern (3-3-3-3 and feeder with cross-bussing) significantly improves the racial balance for Negro and Oriental measures, it has little effect on Negro-White integration. Therefore unless (sic) the District wants to cross-bus Negroes and Orientals, cross-bus whites into lower socio-economic neighborhoods and radically modify school facilities, it appears that the level of racial balance attainable in the C-2, C-3, C-4 or C-5 patterns (3-3-3-3 and feeder) is about as much as can be realized."

One hesitates to be critical of any sincere effort to solve urban school segregation problems, but the solutions resulting from this study seem less than adequate. In the event, none has yet been selected for implementation.

Across the Bay in Berkeley, however, another study of the alternative methods of pupil-assignment will bear fruit in September 1968[3]. Berkeley is a relatively small city (120,000) and has effectively only one minority group, but the decision has been taken to bus all elementary school children at some stage in their school careers. Elementary schools will be either K-3 or 4-6, so that, with some modification, existing plant can be used. The plan also includes a re-assignment of teachers to attain equivalence of staff.
There are two educational aspects of schooling, integrated or otherwise, that need to be mentioned here. The first concerns our great striving for "excellence" or "quality" in education, and the second, which may be more closely allied to the first than we often admit, is the influence of factors other than the school itself. To take the second aspect first: We are today aware that education and educational opportunity extend beyond the simple teacher-pupil relationship. The enculturation process of the peer group is becoming more fully recognized and accepted as an integral part of the educational process. While there are honest differences of opinion among educators about the role played by each student's associates, the fact that student-student interaction materially affects achievement is no longer seriously questioned. Recent statistical studies[4] indicate that the factors which contribute most to variations in student achievement for minority-group children are those related to the educational background and aspirations of other students in the school. The analysis shows that the achievement of minority-group children increases significantly when their classmates have strong educational background and motivation. Another study[5] has shown that the earlier a minority-group student attends a mixed school, the higher his grade-level performance. Negro children in the ninth grade who first attended mixed schools in Grades 1, 2 or 3 performed 1½ grade-levels higher than ninth-graders who had never attended a mixed school. Thus there are grounds for expecting a net national gain in educational achievement by increasing the extent of integration and, further, for supposing the gain to increase as we lower the age and grade-level at which integration is achieved. We have quite recent evidence of this from a study carried out in White Plains, New York. Integration not only raises the overall achievement level of minority and low-income groups but it does not significantly affect the achievement of majority and higher-income group children. Further, the evidence is now coming in that our belief in early integration was correct. Emphasis must be placed on the K-4 grades in all schools. By the end of fourth grade, the achievement pattern and life-style of the child are established. Success in matching the child's achievement to his potential in the early school years is, as we all know, vital to his development in the later educational process. This is, of course, as true of minority-group children as of others, and can best be achieved by close contact with those others.

Excellence in education, except where affected by the above consideration, is not directly dealt with by the mathematical programming technique discussed in this paper. In our approach, we shall accept the premises that school facilities in a district are already equivalent, and that a school authority will staff all schools of a given type to the same qualification level. Certainly, the technique can be applied to determine the allocation of teachers, by qualification or by race, which comes nearest to producing equality of staff and standards. The added complexity of considering staff allocation jointly with pupil assignment will, however, not really produce any offsetting advantages. There is, of course, a real connection (which we often try to gloss over) between excellence and integration. The positive side of the picture concerns the improved motivation, ability, and achievement of children who were previously served inadequately. The negative side concerns financial outlay. Every program initiated to achieve integration costs money, and money spent on buses, say, cannot be spent on teachers' salaries, tape-recorders or laboratory equipment. Most School Boards are already in a quandary caused by rising costs and cannot always be blamed for feeling that the choice is between integration and excellence. Faced with, one might almost say confronted by, groups of parents with widely conflicting aims, they struggle to do their jobs. Some rational bases for
decision-making are badly needed to replace the heated arguments, claims and counter-
claims made by organized or disorganized groups in what often becomes a highly-charged
atmosphere.

Let us therefore examine in detail what mathematical programming can do in this
situation. We propose to contribute to understanding the needs of public-school systems
by applying analytical methods to the task of pupil assignment within a school district.
Mathematical programming has been a powerful aid in business and military operations
and economic planning. It is a proved technique for assigning resources (say school-age
children) to facilities (say schools), subject to restrictions on facilities (say school capacity
limits) and resources (say a maximum travel time, or a desirable range of school ethnic
compositions), so that a measure of performance, the “objective function” (say total cost
in dollars, or total time of travel) is optimized (that is, maximized or minimized, depending
on the parameter chosen).

While this analytical method can provide theoretically optimal solutions to the problem,
we do not expect any solution we obtain to be optimal in a real sense. To achieve a truly
optimal solution, we require an objective function which represents the true values of each
unit of allocation. In the problem of student assignment, there is no single objective function
which would represent the views of all educators, school administrators, or judges, let
alone all citizens! It is highly likely that no consistent “consensus” objective function can
be developed. If different solutions are calculated for a variety of objective functions and
for various sets of constraints, however, each solution represents in theory the logical
culmination of a point of view, an attitude, or a policy. Implementation of any particular
policy may then still have to be the result of legislative or administrative discussion, but
such discussion can then center on the consequences of a course of action rather than on
any individual point of view.

Many of the conflicts involved in assigning pupils and teachers, and in school allocation,
can be resolved by the use of the mathematical programming technique because the solution
to each problem can be calculated in advance, giving factual data that are trusted by all
participants. Consider these examples:

(1) By careful selection of school segregation indices (based perhaps on residential
and family-income distributions), we can determine whether the steps taken to comply
with court decisions have been merely token ones, or whether they fall within the range of
reasonable responses to such court decisions.

(2) The degree of integration attainable within a given budget for student travel can
be determined.

(3) Alternatively, we can determine the travel costs required to obtain a given range of
an integration index.

(4) The technique can be used to examine the geographical districts for neighborhood
schools in a school system, given the distribution of population, and thus obtain measures
of integration attainable without major changes in school-area boundaries.

The results for all four of the problems just listed can be obtained for a school district
from the same basic set of input data; only the boundary constraints and the objective
functions need to be modified to answer each question. As a technique, mathematical
programming is well known for allowing a large degree of flexibility at low cost. Certainly
the cost for collecting the input data is not negligible and, several decades ago, this might
have proved a considerable obstacle. Recently, however, many School Boards have been forced, by the public and legal attention paid to integration problems, to collect and verify the very input data which are used in the mathematical programming model. One might almost say that it would be a waste of money not to use the data, soon to be widely available, in this way and thus extract the last ounce of usefulness from them!

It is not so long ago that a mathematical programming model for resource allocation had to be limited in size if the computational effort required for feasible solutions had to be kept within reasonable computer time and budget limitations. The coming of the third generation computer has now brought even large problems (several thousand variables) within the realms of possibility. Results can be obtained fast, and a few minutes or even a few hours of computer time are hardly budget-breaking. The objective of the mathematical programming approach is to develop a mathematical model of pupil assignment, which is flexible enough to allow examination of a wide range of alternative objective functions and system constraints, and is easy to use and to interpret. One needs to avoid the situation in which “the computer” is regarded as telling people what to do. Where the education of their children is concerned, parents and School Boards, educators and administrators should all feel that they gave the computer their own objectives and that the machine merely provided them with an accurate solution more quickly than they could have derived it themselves. The initial model should be capable of analyzing school systems for metropolitan areas with school populations of up to about 120,000 children and about two hundred schools. Once formulated, there is of course no obstacle to applying the same model to smaller problems.

The problem of neighborhood school assignment is the classical “transportation problem” of linear programming. Computer programs and techniques for rapid computation already exist and are widely used by industry and commerce. Once developed, the modeling technique and methodology for school assignment can readily be used by School Boards and their consultants in all parts of the country. With the development of time-sharing computers and networks of remote consoles, no urban school district now is far from a large computer.

As an example of how the technique can be used, we can consider the selection of attendance areas for neighborhood schools. One interpretation of equity in determining the boundaries of attendance areas is that the areas should be “compact and contiguous.” A mathematical definition of compactness would imply that the sum of the product of the squares of the distance from each census enumeration district to the assigned school for that district and the number of children in the district would be a minimum. Contiguity implies that there are no isolated districts surrounded completely by other districts assigned to a different school. Given these definitions, the number of children in each age group in each enumeration district, the capacities of the schools, and the distance from each enumeration district to each school, we can determine the attendance areas.

The solution obtained in this way may not be acceptable. For example, it may require students to cross major traffic arteries or to travel unduly long distances. Further constraining equations can be added to take account of these factors, and a new solution can be obtained. New facilities and impending population changes can be accounted for by changing the school distances and the number of students in the enumeration district.

All this has not yet included the racial composition of the enumeration districts, or constraints on the racial composition of the individual schools. Using the U.S. Census data (or school census data), we can determine the racial composition of the individual
schools that is implied by the solution obtained, and compute a school segregation index.

Using the same census data on racial composition of the enumeration districts, with a set of constraints on the range of racial composition desired in the schools, we can obtain a new solution for the attendance areas—provided a solution exists. It is highly likely that no assignment using the criterion of compactness and contiguity will meet the desired range of school population by racial composition; that is, there will be no feasible solution. If that is so, neighborhood schools and decentralization of school districts may not comply with legislative acts or with the governing constitutional principles. Alternative solutions—and alternative objective functions—must be sought and tested.

For input data, we shall assume that data on children of school age are organized by the smallest population unit designated by the Bureau of the Census: the census enumeration district. For each district, then, we have the number of children in each age group classified by race, the distribution of family income, and a breakdown of the years of completed education for the district as a whole. Although the 1960 census no longer provides us with "hard" data on some crucial matters, the model, when formulated, is ideally constructed to use the 1970 data immediately they become available. Other data on the children, such as achievement distributions, must be obtained from a school census. Such data are now available in many school districts, although they have not always been made public.

We next assume that we know the location of each school and its capacity for each grade group. The latter consideration is not a "hard" fact either, but may be a function of policy (as when the capacity of a particular classroom changes because two K-6 schools become one K-3 and one 4-6 school). Alternative capacity figures can be obtained, knowing the number of rooms used as classrooms and the class size desired in the rooms. An independent estimate of capacity would also be desirable as an alternative, particularly where present overcrowding practice is so far from ideal that it should not be perpetuated. The travel distance or the travel time from each district to each school are known or can readily be computed, while realistic cost accounting figures give travel costs for busses or other means of transportation.

The mathematical formulation is intrinsically simple. Consider a system which has $I$ schools, and $J$ census enumeration districts. Within the school district, there are $K$ different minority groups, and children are categorized by $H$ age-groups.

Let $i$ indicate a school \( i = 1, 2, \ldots, I \)

$\text{j}$ indicate an enumeration district \( j = 1, 2, \ldots, J \)

$k$ indicate a minority group \( k = 1, 2, \ldots, K \)

$h$ indicate an age-group \( h = 1, 2, \ldots, H \)

There are $N_{jkh}$ children in enumeration district $j$, of minority group $k$ in age-group $h$. Thus,

$$\sum_{h} \sum_{k} N_{jkh} = \text{total school population in enumeration district } j$$

If we assume that the school population is uniformly distributed over district $j$, then we can consider the center of gravity of the district as the point chosen for measuring distances and travel times.
The assignment of children to schools is denoted by the variables $X_{ijkh}$, where $X_{ijkh} =$ number of children of age-group $h$ and minority group $k$ residing in enumeration district $j$ and assigned to school $i$.

We can define the movement characteristics by

- $t_{ij} =$ travel time from district $j$ to school $i$
- $d_{ij} =$ travel distance from district $j$ to school $i$
- $c_{ij} =$ cost of moving one pupil from district $j$ to school $i$.

The school characteristics are defined by

- $r_{ih} =$ capacity of school $i$ for age-group $h$

or

- $R_i =$ capacity of school $i$

The basic constraint equations can then be written as

1. \[ \sum_k X_{ijkh} \leq r_{ih} \quad i = 1, 2, \ldots, I; \quad h = 1, 2, \ldots, H; \]

that is, the number of children in an age-group assigned to a school cannot exceed the school’s capacity for the age-group; and

2. \[ \sum_i X_{ijkh} = N_{jkh} \]

that is, every child is assigned to a school; and

3. \[ X_{ijkh} \geq 0 \quad \text{for all } i, j, k, \text{ and } h. \]

In the simplest method of determining attendance areas for neighborhood schools, we could use “compactness and contiguity” of the areas as an objective function. Using the concept of “moment of inertia,” compactness is achieved when the sum of the squares of the weighted distances from districts to assigned schools is minimized[7]. We weight each distance term by the number of children assigned who move the distance. Hence we would have

4. \[ Z = \sum_i \sum_j \sum_k \sum_h X_{ijkh} (d_{ij})^2 \]

as our objective function to be minimized. (Although the objective function is quadratic in $d_{ij}$ it poses no difficulty, for the $d_{ij}$ are known values and the equation is linear in the variables, the $X_{ijkh}$.) We could also have used $t_{ij}$ in place of $(d_{ij})^2$ and would then have obtained the allocation which minimizes total travel time. If there are costs associated with travel, we might replace $(d_{ij})^2$ by $c_{ij}$ and perhaps restrict the value of the maximum travel time. For urban schools, the “compactness” measure would be more appropriate.

We have not yet included any measure of integration. Now let us define the minimum and maximum fraction of minority groups in a school. We will here assume only two groups, white and non-white. Let $k = 1$ denote non-white. Then

- $a_{it} =$ minimum fraction of non-whites desired in school $i$
- $b_{it} =$ maximum fraction of non-whites desired in school $i$

Then

5. \[ a_{it} \sum_j X_{ijkh} \leq \sum_j X_{ijkh} \leq b_{it} \sum_k X_{ijkh} \]
Since we require that

\[ \sum_{j} \sum_{k} X_{ijkh} \leq r_{ih} \]

we have two more equations for each school i.

\[ (6) \quad a_{it} \sum_{j} \sum_{k} X_{ijkh} - \sum_{j} X_{ijth} \leq 0 \]

\[ \sum_{j} X_{ijth} - b_{it} \sum_{j} \sum_{k} X_{ijkh} \leq 0 \]

Introducing these constraints will change our solution (the values of \( X_{ijkh} \) in minimizing equation 4). It may, in fact, produce districts which are not contiguous; they will certainly be less compact. It will also increase the travel time and the cost of travel.

Alternately, we could fix a travel budget, that is, set

\[ (7) \quad \sum_{i} \sum_{j} \sum_{k} c_{ij} X_{ijkh} = F \]

and seek the values of the coefficient \( a_{it} \) in equation 6, which is now a variable indicating the degree of integration that could be obtained, by optimizing the compactness equation (equation 4).

Other constraints implied by a policy or point of view can also be formulated and examined. Similarly, other objective functions are possible; for example, one might wish to set all schools in a given region at the same degree of utilization. All possible formulations cannot be included in this brief examination. We hope that it will be possible to include other student characteristics (for example, achievement distributions) in the computation. If such data are available by geographic area, their inclusion is straightforward.

It is important to realize that the technique can be used to simulate many different methods of school assignment, not all of them practical by any means, just to compare results. In the Netherlands, for example, children are assigned to schools on a random or semi-random basis, independent of the residence area of the pupil. If this would be a preferable method, the solutions can be used to provide the probabilities of allocation from a given area to a school; that is, the weightings to be given to each assignment probability. Certainly, such proposals as "educational parks," or the recently published "decentralization" plan for New York City schools, which are now being so much discussed, can be evaluated. It is interesting that two conflicting theories now seem to be gaining favor at the same time. Both advocates of "centralization" and of "decentralization" are convinced that they have the solution. In each case a number of objective functions and constraint sets would be tested and the results of each test can be presented to the School Board, legislative body, or court in the form of maps representing the best possible set of attendance areas under the conditions given. Discussion and evaluation is then based on the consequences of applying the rules and guidelines used, and not on the rules and guidelines themselves.

In summary, therefore, the mathematical model is intended to assist School Boards in assignment and planning, legislative bodies in developing guide rules and regulations for school districts, and the courts in evaluating compliance with constitutional principles. Some of the responsibilities of School Boards and the courts have been clearly expressed in Brown versus Board of Education of Topeka (349 U.S. 294; 1955)[8].
Mathematical programming can provide a technique and a methodology to aid in implementing the governing constitutional principles within a School Board's jurisdiction (or between School Boards if regional fragmentation is used as a device to avoid the intent of the court), taking into account the reconciliation of public and private needs.

Much as the approach and techniques described here may do, there are some things they will not do. If the schools are to continue to carry the brunt of the battle for integration while residential, economic, and social conditions are ignored, the task may never be completed. The success of integrated school systems, measured in terms of student achievement, student attainments, and improved social relations, can contribute to an extension of integration to residential patterns and to equality of economic opportunity, and that is where our hopes must lie.

REFERENCES
8. The previous Brown versus Board of Education citation (347 U.S. 483, 1954) was the decision on the merits of the case. The citation above is on the decree necessary to enforce the decision of 1954.