Book Review: Adding depth to portraits of mathematical inquiry
Karin Brodie (2010) Teaching mathematical reasoning in secondary school classrooms
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In her new book, Brodie surveys the methods and dilemmas involved in promoting mathematical reasoning among students who bring and have access to different kinds of resources for their mathematics learning. Drawing upon first-hand accounts of teacher researchers who worked collaboratively with her to cultivate productive mathematical inquiry in their classrooms, Brodie offers a framework that specifies the functioning of particular teacher moves in response to different types of student contributions. Brodie’s account is deeply grounded in the intricacies of moment-to-moment classroom interaction and in the accomplishments and tensions that emerge within complex classroom mathematical activity systems. Her book is crafted in such a way that it is while written primarily for mathematics educators, it also speaks eloquently to teachers through the case studies, and to policymakers through concise and coherent summaries. In this review, I focus on how the structure of the book, and its developments along the way, evolve into a strong narrative around the intricate work of teaching mathematical reasoning and the importance of developing more sophisticated and authentic tools for supporting secondary mathematics teachers in this work.

1 Prioritizing mathematical reasoning

Reform initiatives for mathematics education around the world coalesce around a vision of mathematics teaching as involving learners in reasoning substantively about foundational ideas in mathematics. The government of South Africa is no exception. The post-apartheid government has instituted a set of education reforms aimed at preparing all students to participate generatively in the new South African society. This may be a tall order for a country that faced (and continues to face) such profound inequity between white and black people. Brodie posits, however, that the new mathematics curriculum, which draws heavily
from reform initiatives underway in other countries, may serve as a vehicle for bringing about greater participation among a range of students in mathematics learning, and thus society more broadly. For the new vision of mathematics education introduced in South Africa to serve in this capacity, however, reform has to be widespread—reaching into townships and other less-resourced areas that often do not receive the support they need to cultivate fertile ground for educational change. From Brodie’s perspective, well-equipped teachers are a critical part of this support. To some extent, I share her belief (as do many other researchers in the teacher education community) that teachers play an important role in implementing and sustaining reform. However, I argue in this review that this underestimates the role of broader structural forces and processes that play out in everyday moments of classroom mathematical activity. That said, Brodie is very specific about what she means by equipping teaching in ways that will help them progress towards reform ideals. These are primarily to provide accounts of the struggles and triumphs that teachers have experienced in shifting their practice towards engaging their learners in productive and expansive mathematical reasoning.

2 Privileging and scrutinizing teacher practice

The book is an account from the perspective of Brodie and five secondary mathematics teachers with whom she collaborated. In part, it is structured around the teachers’ stories, which is compelling for several reasons.

The first reason is that the book has a surprise ending. The conditions under which each teacher is working represent stark contrasts in educational resources, mirroring the current situation in South Africa. One of the teachers, Ms. King, teaches in a safe, lush, and serene environment, where students are housed in dormitories and play on well-tended sports fields. Her classroom is newly remodeled and has ample resources for her students and for her instructional development. Twenty-seven 10th graders with strong mathematical knowledge populate her high-track mathematics class. In contrast, Mr. Peters works in a school in disrepair, where electricity is not available, and where there are not enough textbooks to go around. There are an unheard of 45 students assigned to his 10th-grade untracked mathematics class. Given these extremes, I anticipated hearing markedly different stories about these classrooms. In fact, the opposite was true. Although school resources were highly correlated with student socioeconomic status, and additionally with students’ mathematical knowledge, links to these and the nature of the mathematical reasoning that emerged across the classrooms was not so straightforward. For example, highly resourced schools did not necessarily have more students constructing strong mathematical arguments. The level of mathematical reasoning in a classroom was much more intertwined with the structure of the mathematical tasks, teachers’ pedagogical moves, as well as students’ mathematical understanding.

A second reason for the freshness of these accounts is the extent to which they lay bare the teacher’s practice. This level of transparency is uncommon in the mathematics education literature, where researchers often have to infer a teacher’s intentions, planning, strategy, and assessment. In this book, in which each teacher contributes a chapter reporting on their action research project, instructional moves and students’ responses are meticulously analyzed to make sense of the opportunities for mathematical reasoning afforded to classroom participants. For example, Mr. Nkomo illustrates his struggle in attempting to generate robust classroom discussion around a mathematical explanation offered by one of his students and generates several reasoned and insightful explanations.
for this situation. Mr. Mogale analyzes the ebb and flow of his teaching, where at times he managed to facilitate student-to-student conversations and at others he relied on a didactic approach. Through their accounts, the teachers convey the complexity involved in making rapid instructional decisions. It is difficult to do justice to the chapters authored by these teachers (Mr. Mogale, Mr. Peters, Ms. King, Mr. Nkomo, and Mr. Daniels). Instead of reviewing each one separately, I hope to let their voices and teaching practice resonate throughout this paper.

For teachers to reveal their struggles and frustrations is an act of courage. One wonders how Brodie was able to cultivate this deep sense of trust among her teacher researchers. One reason could be that the teachers she selected are unique in their ability and desire to reflect critically on their practice. Another could be the teachers’ sense of solidarity with Brodie towards enacting the critical reforms laid out by the South African government. To me, Brodie’s fair and balanced approach to the analysis of mathematics teaching may have also helped her gain credibility. Her aim to delve into the intricacies of reform mathematics teaching is reflected in her remarks in the final chapter of the book, where she calls upon the research community to better acknowledge the “messy” work of learning to teach for mathematical understanding. Brodie’s explanation—that “achievements give rise to new challenges” (p. 200)—once again captures the more fluid and dynamic nature of teaching in complex learning environments (Bransford, Brown, & Cocking, 1999).

How does Brodie account for this messiness in her research? Her approach is twofold. Firstly, she provides a “language of description for teacher and learner talk … where mathematical reasoning is a focus” (p. 199). To accomplish this, she draws on extant literature and her analysis of (a) types of mathematical tasks, (b) learner contributions, and (c) teacher responses to learner contributions. Once these coding schemes are in place, Brodie then analyzes patterns in the relation between these classroom features and the mathematical reasoning that emerged within and across the classrooms. Within these patterns she also identifies dilemmas and tensions that these teachers faced and that push on our understandings of best practices of reform mathematics teaching. The framework, patterns, and dilemmas are explored below.

3 Developing a solid framework for teaching for mathematical reasoning

Mathematical tasks Brodie’s first step in analyzing the features of reasoning that occurred in each mathematics classroom was to explore the affordances of the mathematical tasks. These tasks were developed collaboratively by the teachers who split up by grade level (10th and 11th grades). Drawing on the Stein, Grover, Henningsen, and Henningsen (1996) framework of the levels of cognitive demand of mathematical tasks, Brodie carefully details the support each task provides for students’ mathematical reasoning. For example, the task for the 11th-grade students involved shifting parabolas along the x- and y-axes and reasoning about the changes produced in the quadratic equations. Brodie categorized this task as affording “procedures with connections”—meaning that the activities encourage learners to make sense of mathematical procedures in order to gain a deeper understanding of mathematical concepts and ideas. The 10th-grade task revolved around making conjectures about the behavior of mathematical functions (e.g., $x^2+1$ can never be 0). In this case, Brodie illustrates an important distinction between the intended activity supported by the task versus the way that learners take it up. For example, on a worksheet tailored by
Ms. King, one of the activities could support students in solidifying their understanding of the meaning of function notation (“procedures with connections”), or in simplifying algebraic expressions (“procedures without connections”). The important point being that while tasks may support a form of mathematical activity that we value, this activity may not transpire within the classroom.

Learner contributions The second part of the framework was categorizing student contributions in class discussions (as captured by transcripts of classroom interaction). Brodie identifies six different kinds of contributions, grouped according to whether they represent some form of correct response, or learner error. The correct responses are as follows: Partial insights are contributions that point to an important mathematical idea but are incomplete or slightly incoherent. Complete responses are just that, and those that go beyond the task signify that the learner has extended her reasoning beyond the task requirements (e.g., making unanticipated connections or generalizations). Learner errors are laid out in similar fashion. There are errors that she categorizes as basic, or below grade level, and there are appropriate errors and missing information errors, both of which are expected, and represent either an alternative conception, or a potential solution with limited information.

The relation between learner contributions and the nature of the task, teacher pedagogy, and learner knowledge, is considered next. Brodie identifies several interesting patterns here. Basic errors were only prevalent in one classroom, which Brodie links to the students’ extremely weak mathematical background and, interestingly, to the teacher’s pedagogical focus which centered on unearthing learner errors and prompting learners to engage in reasoning around them (Mr. Peters). This is contrasted with Mr. Nkomo’s class, also comprised of less proficient mathematics learners, but where the teaching moves did not afford error identification, and thus, in which basic errors did not emerge. So, while we would expect mathematical knowing to be related to the number of basic errors made by learners, teachers’ pedagogical practices have more to do with whether and how these errors show up.

Contributions that involved missing information or partial insights were interesting cases since these responses are characteristic of inquiry-oriented mathematics classrooms. Teacher moves, then, play a prominent role, but in different ways. Brodie illustrates how some teachers tended to support a rather narrow pattern of response giving, where contributions were explicitly funneled toward the construction of a complete solution. In these classrooms, partial insights were less likely to occur. Two of the teachers in her study (Mr. Daniels and Mr. Mogale) fostered more student-to-student interaction which supported a higher level and broader range of partial insights. However, these did not necessarily result in complete responses. Brodie argues strongly that both kinds pedagogical moves (i.e., teacher-directed and student-centered) play an important role in the development of mathematical reasoning.

As suggested above, complete responses were likely to occur in classrooms where the teachers strongly guided students towards these insights, either to move quickly through learner errors, or to establish something in the classroom common ground. Learner responses that go beyond the task were rare. Given the data that Brodie presents, I would hypothesize that this response has to do with a disposition learners bring to their learning to push for deeper sensemaking, and the opportunities created within a learning environment for students to engage in this practice. This may be a fruitful direction for Brodie to pursue in future research.
4 Teacher responses to learners’ errors

One of the ways that learners’ responses become transformed into productive or less productive classroom activity is in the way they are positioned through teacher’s pedagogical moves. Responses that are ignored or treated as off-task send a message to students that they are not participating competently in the classroom. These moves are likely to curtail further participation by these students. Contributions that are followed up, as Brodie describes, can position students and the mathematics they are working on in a variety of ways. She categorizes teacher follow-up responses into five types, describes how they relate to and extend prior work in the field, and examines how they are employed by teachers to work with student contributions.

Two of the responses serve to push on or extend students’ mathematical reasoning. These include the elicit and press moves, where a teacher is either trying to solicit new information from the students, or to push them to elaborate or strengthen their ideas. These moves were commonplace in Mr. Mogale’s teaching, who, as described above, attempted to distribute authority to students to make sense of the mathematics together. They were also characteristic of the instructional practice of Mr. Peters, that was centered on exposing and confronting students’ errors in explicit ways.

The other set of responses involves teacher interjections, which are aimed at controlling the trajectory of a mathematical discussion. These include moves to insert an idea, solution or link, to maintain a contribution by focusing or re-focusing attention on it, and to confirm the meaning of a learner contribution. All of the teachers were likely to engage in some level of inserting and eliciting moves, and less likely to spend time confirming students’ responses. While these statistics helped to paint each classroom in broad strokes, it was the constellations of teacher moves that proved most revealing to Brodie’s analysis.

For example, she describes how one of the teachers, Mr. Nkomo, relied heavily on the maintain move (nearly 50% of his follow-ups) to attempt to support his learners in focusing on and responding to the errors they were making. This type of teacher move is quite common in classrooms where a teacher is attempting to shift the sensemaking to students but has not yet developed a repertoire of moves to facilitate this process. It points to a possible developmental trajectory for learning to teach in this way.

Another teacher, Mr. Daniels, also had a significant amount of learner errors in his classroom. He, too, focused on these errors, and engaged in the maintain move a high percentage of the time (42%), although at different times and for different purposes. In his case, it was often used to re-voice students’ contributions or to re-engage students into a discussion. After foregrounding a particular piece of mathematics with this move, Mr. Daniels would then press students to delve further into the mathematical rationale behind it. This coupling of moves enhanced the flow of mathematical conversation; however, it did not necessarily result in complete and correct insights.

Through these accounts, Brodie illustrates quite convincingly that the mathematical insights that emerged within and across the conversations were bound up in a range of classroom features. However, the features she examines here are not exhaustive, and recent research on issues of equity in mathematics classrooms suggests that important ones are missed. I am confident that, given the time and space, Brodie would have included additional features in this analysis (e.g., teachers moves around forms of students’ everyday mathematical reasoning; Rodriguez & Kitchen, 2005; and disrupting issues of status that are reproduced through dominant power structures; DiME, 2007). Although these case studies are deeply comprehensive, I believe that they do not adequately capture the range of features that support and constrain classroom mathematical discussions.
What does it mean for the readers that this analysis is incomplete? Brodie would argue that this is precisely her point; that the enterprise of teaching is neither entirely unpredictable, nor is it static. It is apropos, then, that Brodie summarizes her research by discussing the tensions/dilemmas that emerged for the teachers in her study.

5 Embracing tensions and dilemmas

As any teacher working to foster mathematical reasoning through classroom discussion knows, the process can be frustrating. Students’ contributions may build on each other in ways that take the conversation far afield from the teacher’s aims. And it may be difficult to decide when to pursue a contribution, and when to bring the conversation back to the floor. These two teaching dilemmas—what Brodie calls “linking learners with the subject” and “working simultaneously with individuals and groups”—are prominent in the literature on reform-oriented mathematics teaching and stood out to her in these cases as well.

One teacher move highly favored by the mathematics teacher education community is the *press* move, which pushes students to clarify or extend their reasoning, to provide support for their conjectures, or to generate insights that require a deeper understanding of the mathematics at hand. Another practice widely touted in the discourse around reform is inviting a range of mathematical contributions from learners. However, as this research indicates, both practices can meet with mixed results. Brodie focuses on two of the teachers in her study to make this point.

The first teacher, Mr. Peters, was interested in probing students’ ideas for mathematical sensemaking. The distribution of his follow-up moves was heavily weighted towards the *elicit*, *press*, and *insert* categories, with the intention that the elaboration of a particular idea would help the class progress. This was met with more or less success. In one excerpt, we see Mr. Peters repeatedly attempt to extract a line of mathematical reasoning from his student, only to have the student supply short and terse responses to his questions. Arguably, he eventually succeeds, only to reflect later that the process may have made a rather shy student uncomfortable, and limited broader class discussion. This is a predicament in which many skilled mathematics teachers, such as Mr. Peters, who attempt reform pedagogy may find themselves. And, as Brodie convincingly argues, it deserves more nuanced treatment by the mathematics education research community.

The second teacher she highlights, Mr. Daniels, faced a different dilemma in attempting to elicit an assortment of learners’ contributions in class discussions. In this case, the tension that arose was less related to helping a particular learner successfully navigate a path within the mathematical terrain, and more about how to stitch together the mathematical ideas on the table into a coherent mathematical whole. Given the overflow of ideas, Mr. Daniels found that he sometimes ended up ignoring responses that he initially favored. Once again, Brodie’s analysis points to the problematic nature of implementing reform mathematics practices on the ground.

A third characteristic of mathematics teaching that presses learners to work through ideas on their own or publicly, is the emergence of learner resistance. I categorize this as yet another tension, despite the fact that Brodie does not, and agree with her that how forms of resistance differ can have important implications for how we deal with them. One form of resistance relates to broad-scale antipathy towards having to make sense of what is being taught, instead of obtaining unequivocal answers. In my own university courses, I have witnessed less resistance among students over the last 5 years around constructivist teaching methods, which indicates that norms for (mathematical) reasoning are becoming
more prevalent in formal education in the USA. However, Brodie also identifies a second dimension of resistance—which she argues relates back to the dilemmas discussed earlier—in which the teachers faced challenges in attempting to generate mathematical reasoning through their students. Drawing again on the case of Mr. Daniels, she illustrates how learners who were disposed to contribute to classroom inquiry and make sense of mathematics were frustrated by the lack of clarity and closure afforded by circuitous classroom conversation. Contributions by peers that missed important information or provided only partial insight into the solution were understandably confusing. Accordingly, Brodie calls for increased attention to helping teacher frame and authorize learner contributions in ways that create more opportunities for clarity and sensemaking in classroom conversations.

6 Conclusions

We expect a great deal from our mathematics teachers, and it does them a disservice to trivialize the challenging work of learning to teach for mathematical reasoning and the complex nature of classroom mathematical learning environments. As Brodie and her teacher researchers illustrate in this book, attempting to modify one aspect of a classroom practice can provoke a series of micro shifts in other aspects, which can result in unexpected outcomes and tensions. Similarly, enacting a particular teaching move—such as the maintain move—can play out differently across learning environments with different sets of features. If we acknowledge these intricacies and begin to work more closely with mathematics teachers, as Brodie has done here, I believe we will be more likely to motivate the kind of transformative change that we aspire to in mathematics education. It was a pleasure to reflect deeply on Brodie’s work and to begin to envision the powerful role mathematics education will play in bringing about more equitable participation in South African society.

References
